

THE EFFECT OF VARIABILITY IN WAVE HEIGHTS ON THE PROVISION OF DEPENDABLE 'GREEN' POWER: A STATISTICAL ANALYSIS OF NORTH AMERICAN WATERS USING THE GAMMA DISTRIBUTION

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ABSTRACT

The following paper endeavours to analyse the effect of variability in wave heights on the provision of dependable 'green' power. A low level of variability in wave height is desirable and lends itself to a more reliable energy source. Wave height datasets from North American waters are assessed using a univariate gamma distribution and their theoretical dependability quantified in terms of relative power deviation and 'capacity factor'. Geographic trends in energy density and dependability are discussed. In summary, coastal waters adjacent to California and Oregon appear to be well suited as alternate energy sources and generally speaking wave energy presents as an attractive addition to the North American 'green' power market.

INTRODUCTION

Ocean waves, which can be harnessed as sources of renewable energy, are characterized by a spectrum consisting of wave height, period and direction. These characteristics vary over time and depend on solar and lunar forces, temperature gradients, and geographical factors (Beyene & Wilson 2004). The inherent variability of ocean waves is a problem for wave energy developers, as energy markets require dependable (predictable and preferably constant) sources of power (Denniss 2005). Since wave energy conversion systems cannot be continuously operated at their nominal rating, the wave energy sector must address irregularities in wave characteristics by way of careful site selection and the design of energy conversion systems with high part load efficiency.

This paper assesses the natural dependability of US and a limited number of Canadian wave height datasets. Here 'dependability' is quantified in terms of relative power deviation and capacity factor (defined as the predicted ratio of mean power over rated power). Mathematical models utilized in this work were originally developed in Savenkov (2008) and are an extension of the univariate gamma distribution.

The paper is organized as follows. Before moving in to the major body of work, a short overview of wave energy conversion systems and their current state of development is provided. In the next section, mathematical models used to analyse wave height datasets are described in brief. This is followed by tabularized results of the statistical analysis, including a comparison of dataset relative power deviation and capacity factor. Statistical results are then interpreted and promising datasets (those with high theoretical capacity factor and energy density) are identified. Lastly, the power level of these

datasets is estimated and the potential of these sites for energy extraction and grid integration is discussed.

Wave energy conversion systems: an overview

Wave energy conversion is a relatively new method of exploiting renewable energy. There are several types of conversion system being investigated and trialed around the world. Three common methods, categorized as either ‘terminating’, ‘overtopping’ or ‘point absorbing’, will be mentioned here.

Systems utilizing a ‘terminating’ design absorb energy in one direction while restraining (or *terminating*) incoming waves (EPRI 2005). A terminator usually takes the form of an oscillating water column, where rising and falling wave motion is captured inside a compression chamber and converted into electricity using a turbine (Beyene & Wilson 2004). A 500 kW oscillating water column has been developed by Oceanlinx Ltd and deployed in Port Kembla, Australia. The Oceanlinx (formerly Energetech Pty Ltd) design uses a patented variable pitch turbine (see Denniss 2002).

An ‘overtopping’ wave energy conversion system comprises of collection reservoir and structure over which waves spill, the ensuing water pressure can then be used to drive a turbine (EPRI 2005). Wave Dragon ApS (Denmark) has developed a slack-moored system of this kind featuring two reflectors to focus wave energy onto an overtopping ramp and low head set of Kaplan turbines (Hald *et al.* 2002). The company operates a 20 kW prototype site in Danish waters and is currently undertaking a 7 MW project for deployment off the Welsh coast (Hald *et al.* 2002).

A ‘point absorbing’ system makes use of the vertical displacement of waves. In a design by Rhinefrank *et al.* (2006) a buoy moves armature coils relative to an anchored permanent magnet, or vice versa, thereby inducing an electrical potential. While in an indirect design by Houser & Houser (1995)—incorporated into the Finavera AquaBuOY modules—a buoyancy system pressurizes seawater before sending it to a turbine.

MATHEMATICAL MODELS

Wave power

A water wave with height H (trough to crest), water density ρ_w and acting under gravitational acceleration g , has a total energy (per unit surface area) given by (1) (WMO 1998).

$$E = \frac{1}{8} \rho_w g H^2 \quad (J / m^2). \quad (1)$$

The power in a wave is given by (2) and (3) where wave period, crest width and group velocity (incorporating wave period) are represented by T , L and c_g respectively. Expression (3) applies to deep waters only, as it is based on the assumption that the wave propagation velocity is two times the group velocity (WMO 1998).

$$\frac{P}{L} = \frac{1}{8} c_g \rho_w g H^2 \quad (W / m). \quad (2)$$

$$\frac{P}{L} \approx \frac{\rho_w g^2 H^2 T}{32\pi} \quad (W/m). \quad (3)$$

Variations in group velocity can be disregarded as their effect on wave power is only minor (Denniss 2005). Ignoring variations in c_g and re-arranging (2) leads to (4) where bracketed constants are forced to equal 1 by choosing a suitable crest width (L).

$$P = \left(\frac{1}{8} \rho_w g c_g L \right) H^2 \quad (W). \quad (4)$$

The mean, relative deviation, capacity factor, and other statistical properties of convertible wave power can be calculated using the square of wave height.

Univariate gamma distribution

Wave height data ($x = H$) can be fitted to a two parameter gamma probability density function given by (5). This is illustrated in Figure 1. The denominator of (5) is Euler's generalized factorial (6). Parameter n (which is desirably large) and μ are described as the distribution 'shape' and 'scale', respectively.

$$f_G(x; n, \mu) = \left(\frac{\mu^n}{\Gamma(n)} \right) x^{n-1} e^{-\mu x} \quad (0 \leq x \leq \infty) \quad \begin{array}{l} n > 0 \text{ (real)} \\ \mu > 0 \text{ (real)} \end{array} \quad (5)$$

$$\Gamma(n) = \int_0^{\infty} v^{n-1} e^{-v} dv. \quad (6)$$

The first moment about the origin of a distribution is its mean (or first expectation) and, in this case, would represent the mean wave height. However, in order to calculate the mean wave power (height squared) it is necessary to calculate the second order moment. This is accomplished using the generalized (m^{th} order) expression for the expectation of a gamma distribution given by (7).

$$EX_m = \frac{\Gamma(m+n)}{\Gamma(n)\mu^m} \quad m = 1, 2, 3... \quad (7)$$

Following on, the generalized expression for the standard deviation (defined as the square root of variance) of a gamma distribution is given by (8).

$$\sigma_{x_m} = \frac{\sqrt{\Gamma(n) \cdot \Gamma(2m+n) - \Gamma(m+n)^2}}{\Gamma(n)\mu^m} \quad m = 1, 2, 3... \quad (8)$$

The relative deviation of a distribution is defined as the ratio of standard deviation to its associated moment. Therefore, the generalized expression for the relative deviation (ψ_m) of a gamma distribution is given by (9). This property is important when comparing datasets.

$$\psi_m = \frac{\sigma_{x_m}}{EX_m} = \frac{\sqrt{\Gamma(n) \cdot \Gamma(2m+n) - \Gamma(m+n)^2}}{\Gamma(m+n)} \quad m = 1, 2, 3... \quad (9)$$

The relative *power* deviation of a wave height dataset fitted to a gamma distribution is calculated by substituting the gamma shape parameter (n) into (9, with $m = 2$). The relationship between relative deviation and the gamma shape parameter is illustrated in Figure 1. Detailed proof of expressions (7), (8) and (9) is outside the scope of this work, but can be found in Savenkov (2008) and Denniss (2005).

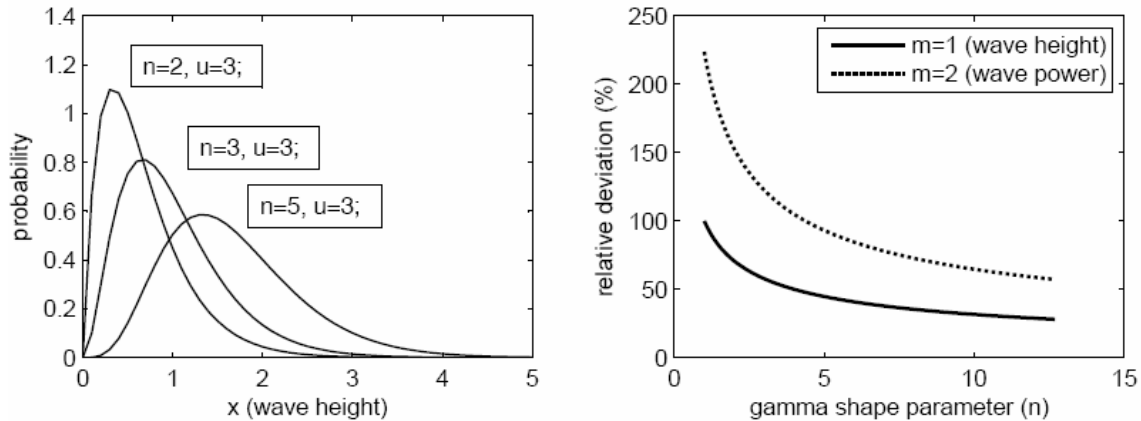


Fig. 1: (left) gamma probability density function, variations in the shape parameter; (right) relative deviation as a function of the gamma shape parameter.

Capacity factor and the ‘10% rule’

Capacity factor is a dimensionless quantity used to describe the ratio of an energy systems average power output over its rated output.

$$CF = \frac{P_{AVG}}{P_R}. \quad (10)$$

The capacity factor of a wave height dataset can be deduced using its relative power deviation and an upper cut-off limit (rated wave height) assigned to its first order cumulative integral. Limiting the cumulative integral from 0 through 90% is known as applying the ‘10% rule’ (Denniss 2005). The distance (γ_r) between the mean of the power distribution and its associated rated point (rated wave height squared) is expressed in units of standard deviation. It can be shown that the theoretical capacity factor of wave height data fitted to gamma distributions, and limited by the ‘10% rule’, is given by (11). For mathematical proof see Savenkov (2008).

$$CF_2 = \frac{P_{AVG}}{P_{AVG}(1 + \psi_2 \gamma_r)} \approx \frac{1}{1 + \psi_2 \left(\frac{((1.34\sqrt{n} + n)^2 - (n+1)n) / (\sqrt{n(2n+2)(2n+3)})}{1} \right)}. \quad (11)$$

While useful when comparing datasets it should be noted that according to (11) all energy below the rated point is converted into electricity at equal levels of efficiency.

RESULTS

Datasets included in the analysis have been chosen with their proximity to coastlines and distance to seabed in mind—this paper focuses on relatively shallow waters suitable for terminating or point absorbing systems. Twenty-five datasets fitting these characteristics have been found describing US waters plus a limited amount to describe

Canadian waters. These sites are spread reasonably evenly, cover a good portion of coastline, and provide a good basis for comparing the levels of energy density and dependability in the region.

US datasets

US coastal wave site descriptions are found in Table 1. All datasets have been sourced from the National Data Buoy Centre (NDBC) a division of the (US) National Oceanic & Atmospheric Administration (NOAA). The average site has a depth to seabed of roughly 120 metres and data totaling 49468 samples observed over 7.24 years.

Tab. 1: US wave height dataset overview

NDBC#	Name	Loc.	Lat.	Lon.	Depth (m)	Obs.	Obs. (years)
44007	<i>Portland</i>	NE	43.53	-70.1	18.9	64913	1982 – 1990
44013	<i>Boston</i>	NE	42.35	-70.7	55	56715	1995 – 2001
44017	<i>Unnamed</i>	NE	40.70	-72.1	44.8	27441	2002 – 2005
44025	<i>Long Island</i>	NE	40.25	-73.2	36.3	63298	1991 – 1998
41004	<i>Edisto</i>	SE	32.50	-79.1	33.5	61786	1995 – 2002
41008	<i>Grays Reef</i>	SE	31.40	-80.9	18	65466	1990 – 2002
41012	<i>St Augustine</i>	SE	30.00	-80.6	38.4	25899	2002 – 2005
41009	<i>Canaveral</i>	SE	28.50	-80.2	41.5	64115	1996 – 1999
42016	<i>Unnamed</i>	WG	30.20	-88.1	-	14463	1988 – 1990
42035	<i>Galveston</i>	WG	29.25	-94.4	13.7	54688	1993 – 1999
42020	<i>Corpus Christi</i>	WG	26.94	-96.7	88.1	51018	1999 – 2005
46087	<i>Neah Bay</i>	NW	48.49	-125	260	12501	2004 – 2005
46041	<i>Cape Elizabeth</i>	NW	47.34	-125	132	57369	1996 – 2004
46010	<i>Unnamed</i>	NW	46.20	-124	-	62909	1980 – 1989
46040	<i>Unnamed</i>	NW	44.80	-124	-	36858	1987 – 1992
46015	<i>Port Orford</i>	NW	42.75	-125	448	22812	2002 – 2005
46027	<i>St Georges</i>	NW	41.85	-124	47.9	59519	1985 – 1993
46014	<i>Pt Arena</i>	NW	39.22	-124	274	59735	1996 – 2002
46013	<i>Bodega Bay</i>	NW	38.23	-123	123	56350	1997 – 2004
46026	<i>San Francisco</i>	NW	37.75	-123	52.1	65493	1989 – 1997
46012	<i>Half Moon Bay</i>	NW	37.36	-123	87.8	48103	1995 – 2002
46011	<i>Santa Maria</i>	SW	34.88	-121	188	59206	1990 – 1999
46051	<i>Harvest Exp.</i>	SW	34.48	-121	-	19899	1992 – 1996
46053	<i>Santa Barb (E)</i>	SW	34.32	-120	417	64988	1998 – 2005
46045	<i>Redondo Beach</i>	SW	33.84	-118	-	61139	1991 – 1999

US wave height histograms have been fitted to gamma density functions. All fit functions have been constrained to retain the same weighted mean as their associated dataset. Their subsequent sum-squared-error (SSE) is given in Table 2. Results indicate wave height distributions are suitably represented by the gamma form.

US site-by-site distribution properties, found using the gamma parameters and the wave energy model, are also presented in Table 2. The average density function has a mean of 1.678 metres, and critically, relative power deviation of 94.1% and theoretical capacity factor of approximately 47%. These results appear to trend according to latitudinal location.

Tab. 2: US wave height dataset density function properties (including capacity factor)

NDBC#	gamma shape	gamma scale	gamma SSE	mean (m)	std. dev. (m)	rel. dev. (%)	E_{avg} (kJ/m ²)	CF (%)
	n	μ		EX_1	σ_{x_1}	ψ_2		
44007	3.0299	3.0878	0.1040	0.981	0.564	121.8	1.610	42.46
44013	2.9711	3.0740	0.2460	0.967	0.561	123.1	1.569	42.31
44017	4.0628	2.7624	0.1102	1.471	0.730	104.0	3.388	44.96
44025	3.9301	3.0156	0.0774	1.303	0.657	105.9	2.678	44.66
41004	5.1972	3.7324	0.1242	1.393	0.611	91.20	2.906	47.30
41008	5.6688	5.3077	0.0756	1.068	0.449	87.10	1.687	48.17
41012	4.3561	3.4724	0.1612	1.255	0.601	100.2	2.432	45.60
41009	4.1349	3.3513	0.0883	1.234	0.607	103.0	2.376	45.12
42016	3.6226	6.1465	0.1108	0.589	0.310	110.6	0.557	43.94
42035	4.4488	4.5513	0.0035	0.978	0.463	99.10	1.471	45.80
42020	4.7230	3.3647	0.0104	1.404	0.646	96.00	3.001	46.37
46087	5.3508	2.6931	0.0136	1.987	0.859	89.80	5.889	47.59
46041	4.1869	1.8444	0.0499	2.270	1.109	102.4	8.024	45.24
46010	3.8408	1.6880	0.0297	2.275	1.161	107.2	8.202	44.46
46040	4.8187	2.0991	0.0614	2.296	1.046	94.90	7.998	46.56
46015	5.4546	2.1706	0.0269	2.513	1.076	88.90	9.392	47.78
46027	5.7884	2.5836	0.0055	2.240	0.931	86.10	7.399	48.38
46014	5.8906	2.2544	0.0096	2.613	1.077	85.30	10.04	48.56
46013	6.1550	2.5349	0.0099	2.428	0.979	83.40	8.614	49.01
46026	6.5715	3.6920	0.0261	1.780	0.694	80.60	4.588	49.69
46012	6.1205	2.7837	0.0298	2.199	0.889	83.60	7.069	48.95
46011	6.4178	2.9814	0.0524	2.153	0.850	81.60	6.732	49.44
46051	7.4873	3.4630	0.0198	2.162	0.790	75.20	6.660	51.08
46053	6.6647	4.8350	0.0227	1.378	0.534	80.00	2.747	49.84
46045	8.0502	7.9031	0.0995	1.019	0.359	72.40	1.466	51.86
Avg:			0.0628	1.678		94.1	4.740	47.01

Canadian datasets

Canadian wave height statistics have been sourced from the Marine Environmental Data Service (MEDS) and are summarized in Tables 3 and 4. The average of these four sites has relative power deviation of 118.9% and consequently a theoretical capacity factor of approximately 43%.

Tab. 3: Canadian wave height dataset overview

MEDS#	Name	Loc.	Lat.	Lon.	Depth (m)	Obs.	Obs. (Period)
C46183	<i>N. Hecate Str</i>	SW	53.57	-131	62	16383	1992 – 1993
C46206	<i>La Perouse Bank</i>	SW	48.83	-126	73	65442	1988 – 1997
C44251	<i>Nickerson Bank</i>	SE	46.44	-53.4	69	42823	1999 – 2004
031	<i>Western Head</i>	SE	44.06	-64.6	40	7410	1970 – 1973

Tab. 4: Canadian wave height dataset density function properties (incl. capacity factor)

MEDS#	gamma shape	gamma scale	gamma SSE	mean (m)	std. dev. (m)	rel. dev. (%)	E_{avg} (kJ/m ²)	CF (%)
	n	μ		EX_1	σ_{x_1}	ψ_2		
C46183	2.2779	1.5923	0.1341	1.431	0.948	142.3	3.702	40.38
C46206	3.9891	1.7098	0.0228	2.333	1.168	105.0	8.557	44.80
C44251	3.8414	1.8659	0.0241	2.059	1.050	107.2	6.714	44.46
031	3.0576	2.3988	0.0843	1.275	0.729	121.2	2.710	42.53
Avg:			0.0663	1.775		118.9	5.421	43.04

INTERPRETATION OF RESULTS

Relationship between the capacity factor and the latitudinal position of dataset is an interesting one. Results spaced incrementally along coastlines trend unambiguously with geographic location. This is illustrated by Figure 2 for combined US and Canadian datasets. Theoretical capacity factor, for both major stretches of coast, is at its highest in southerly locations and lowest in the north. Western coast results (graphs on the left of Figure 2) are particularly poignant, revealing superior levels of predictability in the US south west corner and a linear like decline in sites progressing northward towards Canada. The eastern coast has lower extremes in capacity factor and the trend in results is less linear (probably) due to the wider longitudinal site range.

Generally speaking US wave sources offer excellent potential for energy extraction. A natural economic advantage arises from low levels of relative deviation in wave power, which leads to a high level of dependability. Put simply, US wave heights are fairly ‘tight’ around their mean. South-west and north-west US coasts are most predictable of the US datasets and given a worldwide analysis of relative deviation would likely rate in one of the lowest brackets.

Theoretical capacity factor and average energy density of each dataset can be normalized for the purposes of establishing a relative regional rank, and effect that variations in wave height have on the overall optimality of wave sources. Figure 3 depicts an atlas of sites ranked with an equal weight given to capacity factor and energy density. Using this kind of combined approach, the most promising sites are given—in order of desirability—by *Pt Arena*, *Bodega Bay*, *Port Orford* and *Harvest Experiment* datasets (NDBC reference 46014, 46013, 46015 and 46051 respectively). These four Western coast sites exist around the 40 degree edge of the ‘roaring’ wind zone.

Commercially viable wave power stations require a minimum average power of 15 kW per metre crest length (Harries *et al.* 2006). The average wave power level along the US west coast is given, somewhat ambiguously, by authors as 10 kW through 40 kW (sometimes higher) per metre crest (compare for example, EPRI 2005, Harries *et al.* 2006, Thorpe 2002). These power levels can be checked very quickly using deep water wave equation (3) and assuming an average wave period of 7.5 seconds (sourced from NDBC and fairly consistent for the west coast). The calculated average power is reported in Table 5 and ranges from approximately 9 kW/m through to almost 60 kW/m crest length.

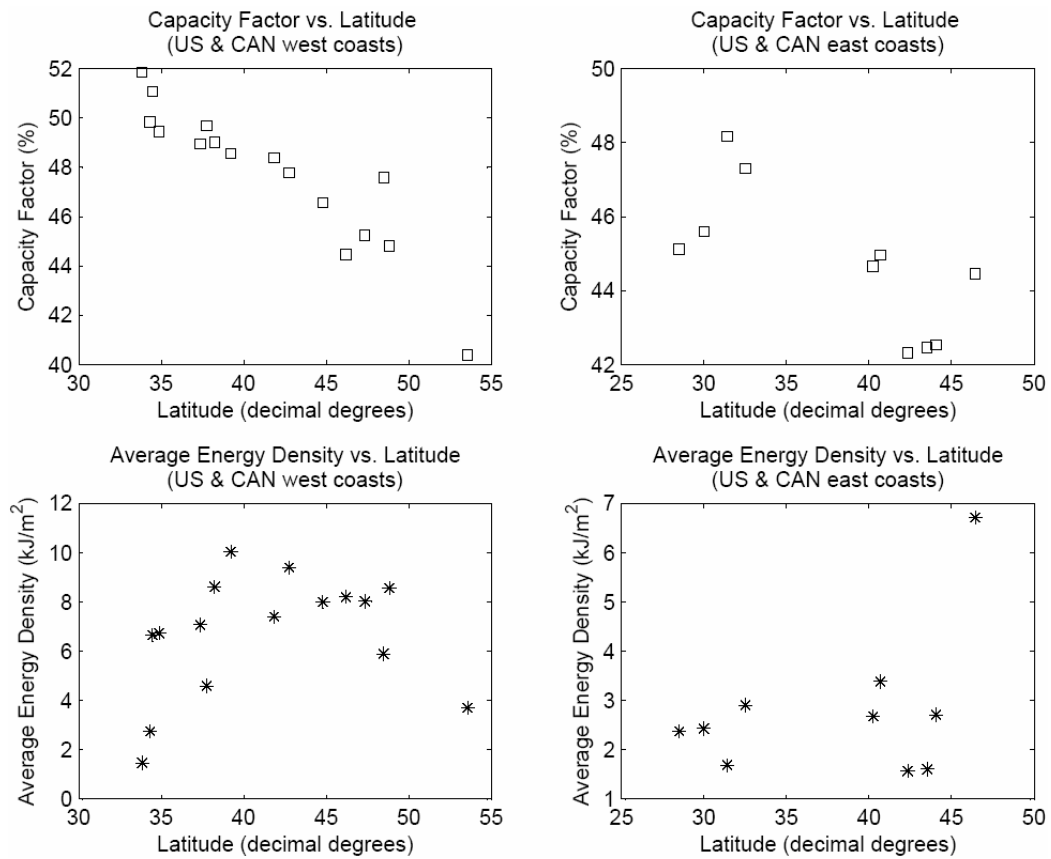


Fig. 2: Capacity Factor and Average Energy Density along the US and Canadian coasts (note: Western coast datasets have longitude range 13 degrees and include NDBC46045 through MEDS C46183; Eastern coast datasets have longitude range 27.5 degrees and include NDBC41009 through MEDS C44251)

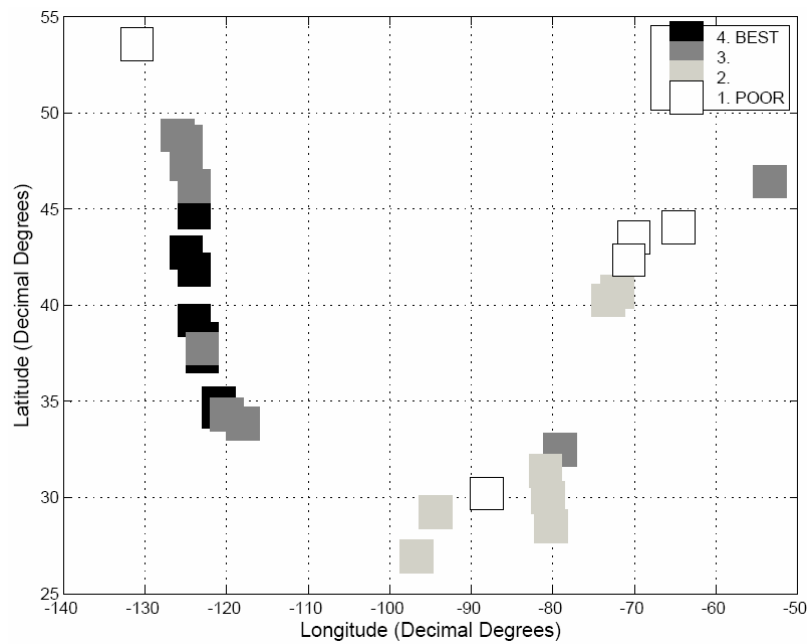


Fig. 3: US and Canadian wave energy sources with rank incorporating capacity factor and energy density

Datasets of particular interest for their high level of energy density and capacity factor (NDBC46013, 46014, 46015 and 46051) appear to have power levels well above the exploitable (15 kW) minima. This is good news for Western coast states within latitudinal range 35 through 45 degrees (including California and Oregon) where there exists a potential for competitively priced wave energy and a naturally high level of dependability in wave source necessary for integration into the wider energy grid.

Tab. 5: Approx average and rated power per metre crest length (US west coast region)

NDBC#	Name	EX ₂ (m ²)	Avg. Period (~s)	Avg. Power (kW/m)	Rated Power (kW/m)
46087	<i>Neah Bay</i>	4.685	7.5	34.480	72.4650
46041	<i>Cape Elizabeth</i>	6.384	7.5	46.980	103.873
46010	<i>Unnamed</i>	6.525	7.5	48.019	108.006
46040	<i>Unnamed</i>	6.363	7.5	46.829	100.582
46015	<i>Port Orford</i>	7.473	7.5	54.992	115.111
46027	<i>St. Georges</i>	5.887	7.5	43.321	89.5310
46014	<i>Pt. Arena</i>	7.986	7.5	58.773	121.065
46013	<i>Bodega Bay</i>	6.854	7.5	50.436	102.936
46026	<i>San Francisco</i>	3.650	7.5	26.862	54.0460
46012	<i>Half Moon Bay</i>	5.624	7.5	41.388	84.5710
46011	<i>Santa Maria</i>	5.356	7.5	39.413	79.7040
46051	<i>Harvest Exp.</i>	5.299	7.5	38.995	76.3490
46053	<i>Santa Barb (E)</i>	2.185	7.5	16.081	32.2680
46045	<i>Redondo Beach</i>	1.166	7.5	8.5840	16.5580

CONCLUSION

North American wave height data has been fitted to univariate gamma distributions. Average energy density, relative deviation, and capacity factor of the corresponding power (height squared) distributions was calculated. The energy density and capacity factor of datasets along each major coastline (east and west) as a function of latitude was graphed. The results were also ranked and presented in atlas form to illustrate the overall optimality (combining energy density and capacity factor) of wave energy sources in the region. It has been shown that the most promising sites, those with a high energy density and high capacity factor, are present on the US west coast in the latitudinal range of 35 through 45 degrees.

Realizing the levels of wave power plant capacity discussed here will undoubtedly prove to be an engineering challenge; that said however, the worldwide wave energy industry is presently undergoing a swift development phase (see Thorpe 2002) and evidence already exists to suggest capacity factors exceeding 40% are practicable (see Denniss *et al.* 2005). Wave energy conversion could therefore play an important role in the emerging North American 'green' power market, and according to the results of this analysis, wave energy conversion is especially well suited to waters adjacent to the states of California and Oregon.

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