

Refined Expressions to Describe the Capacity Factor of Wave Height and Wind Speed Data Fitted to Gamma Distributions

Mark Savenkov

Abstract—This brief paper builds upon an existing method of assessing the relative power deviation and capacity factor of wave height data. Third order power distributions, such as those arising from wind speed data, are incorporated, and refined expressions to describe the capacity factor of both wave height and wind speed data are developed.

Index Terms—gamma pdf, renewable energy, capacity factor.

I. INTRODUCTION

Wave and wind power, like other renewable resources, are inherently intermittent. Wave power is principally a function of wave height squared, and wind power, a function of wind speed cubed [1]. Since energy markets require predictable sources, a minimal degree of variability in wave height and wind speed is desirable. In order to compare prospective energy conversion sites, wave height and wind speed data can be fitted to probability distributions and the dependability of their corresponding power distributions quantified in terms of statistical mean, relative deviation, capacity factor, and so on.

This paper builds upon an existing method of assessing the relative power deviation and capacity factor of wave height data fitted to univariate gamma distributions. The method, first proposed in [2], is extended to incorporate third order power distributions, such as those arising from wind speed data. Then a set of refined, more accurate, expressions to describe the capacity factor of both wave height and wind speed data are developed. To best understand the techniques presented here, the author recommends reading [2] before continuing further.

II. MATHEMATICAL DEVELOPMENT

A. Summary of the existing method

Various studies have shown that wave height data can be represented with good levels of accuracy using a two parameter univariate gamma distribution (see for example [1] and [3]). This form of probability distribution has a density function given by:

$$f_G(x; n, \mu) = \left(\frac{\mu^n}{\Gamma(n)} \right) x^{n-1} e^{-\mu x} \quad (0 \leq x \leq \infty). \quad (1)$$

The relative power deviation of wave height data fitted to (1) is described by the ratio of the second order standard deviation to the second order moment:

$$\psi_2 = \frac{\sqrt{n(2n+2)(2n+3)}}{(n+1)n}. \quad (2)$$

More generally, the relative deviation of any order gamma distribution is expressed as:

$$\psi_m = \frac{\sqrt{\Gamma(n) \cdot \Gamma(2m+n) - \Gamma(m+n)^2}}{\Gamma(m+n)} \quad m = 1, 2, 3... \quad (3)$$

The theoretical capacity factor (defined as the ratio of the mean power to the rated power) of wave height data fitted to (1) can be evaluated by (initially) assigning an upper cut-off limit to the first order (wave height) cumulative integral. The distance (γ_{r1}) between the first order cut-off point and the mean is expressed in units of standard deviation. When a 90% limit is chosen:

$$\gamma_{r1} \approx 1.34 \quad (4)$$

As a result, the capacity factor of wave height data fitted to gamma distributions with a 90% limit is expressed as:

$$CF_2 = \frac{(n+1)n}{(1.34\sqrt{n} + n)^2}. \quad (5)$$

In a generalized form:

$$CF_m = \frac{\Gamma(m+n)}{\Gamma(n)(1.34\sqrt{n} + n)^m} \quad m = 1, 2, 3... \quad (6)$$

B. Extending the method to third order distributions

Wind speed data can also be represented with good levels of accuracy using a gamma distribution (see for example [1]). Because wind power is a function of wind speed cubed, in order to compare the relative power deviation and capacity factor of wind speed data, third order properties of (1) are required. While it is entirely possible to substitute $m = 3$ into the existing generalized expressions (3) and (6), one of the motivations behind this work was to develop simple mathematical expressions that do not rely on the gamma function for calculation of non-integer factorials.

As already alluded to in [2], the third moment about the origin of a gamma distribution (its third expectation) is found using the moment generating function:

$$M(t) = \left(\frac{\mu}{\mu - t} \right)^n \quad (7)$$

The third moment of a gamma distribution is given by the third derivative of (7) at $M(0)$. Hence we have:

$$EX_3 = \frac{(n+2)(n+1)n}{\mu^3}. \quad (8)$$

Following on, the variance (or squared-spread) is:

$$VX_3 = EX_6 - (EX_3)^2 \quad (9)$$

$$= \frac{9n^5 + 72n^4 + 213n^3 + 270n^2 + 120n}{\mu^6} \quad (10)$$

$$= \frac{3n(n+2)(n+1)(3n^2 + 15n + 20)}{\mu^6}. \quad (11)$$

Consequently, the standard deviation is:

$$\sigma_{x_3} = \frac{\sqrt{3n(n+2)(n+1)(3n^2 + 15n + 20)}}{\mu^3}. \quad (12)$$

A comparison of the variability of different distributions necessitates a measure of relative deviation. The relative deviation of a third order distribution is defined as the ratio of third order standard deviation to the third order moment:

$$\psi_3 = \frac{\sigma_{x_3}}{EX_3} \quad (13)$$

$$= \frac{\sqrt{3n(n+2)(n+1)(3n^2 + 15n + 20)}}{(n+2)(n+1)n}. \quad (14)$$

The capacity factor of a wind speed data fitted to (1) can be evaluated in a similar manner to the capacity factor of wave height data. An upper cut-off limit is chosen for the first order (wind speed) cumulative integral. As was previously the case, the distance (γ_{r1}) between first order cut-off point (rated wind speed) and mean is expressed in units of standard deviation:

$$\gamma_{r1} = \frac{S_R - EX_1}{\sigma_{x_1}}. \quad (15)$$

Alternatively:

$$S_R = \frac{\gamma_{r1} \cdot \sqrt{n} + n}{\mu}. \quad (16)$$

Therefore:

$$CF_3 = \frac{EX_3}{S_R^3} \quad (17)$$

$$= \frac{(n+2)(n+1)n}{(\gamma_{r1} \cdot \sqrt{n} + n)^3}. \quad (18)$$

C. Refined expressions to describe capacity factor

A deficiency of the capacity factor expressions developed in [2] stems from the assumption that distance (in standard deviations) between the first order cut-off point and the mean (regardless of parameter values) is approximately constant. However, as distance (γ_{r1}) is a function of distribution shape parameter (n) and the assigned cut-off point, it is possible to refine these capacity factor expressions further.

The author has measured (γ_{r1}) for various cut-off points while using a shape parameter range: ($2 \leq n \leq 14$). This is illustrated in Fig 1 and 2. The range was chosen in accordance with parameter values found by studies of wave and wind height data conducted in [1] and [3]. Typically, wave height and wind speed data has a shape parameter greater than 2 but less than 8, in some extreme cases, such as Hawaiian waters, the shape parameter of wave height is in excess of 8 (see for example [1]).

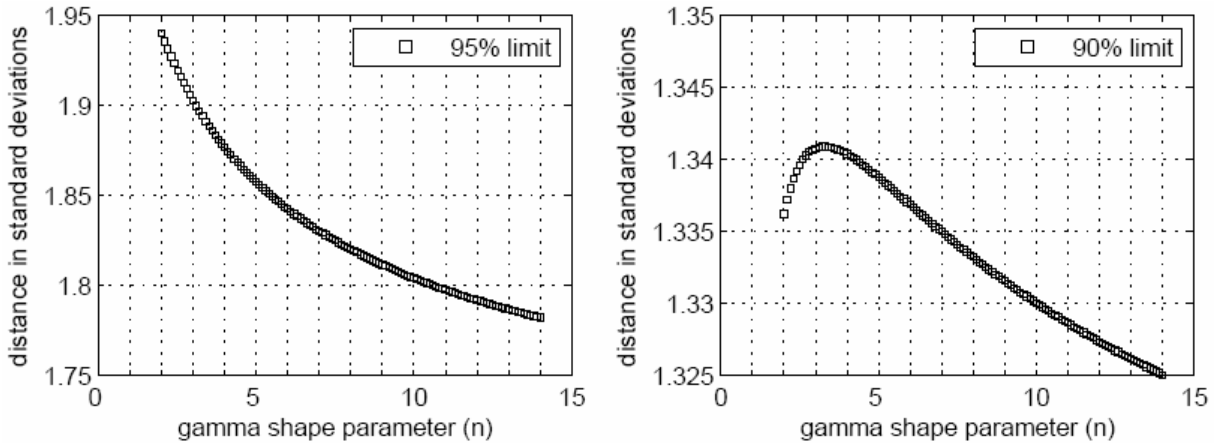


Fig 1. Distance (γ_{r1}) in standard deviations between first order cut-off point and mean, versus gamma shape parameter; (left) power generator runs above its rated capacity on 5% of occasions, (right) on 10% of occasions.

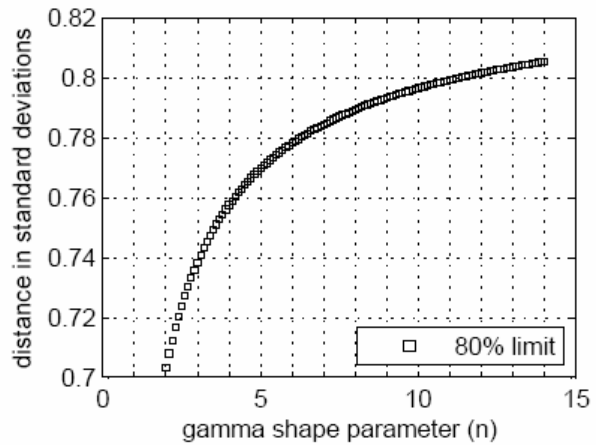
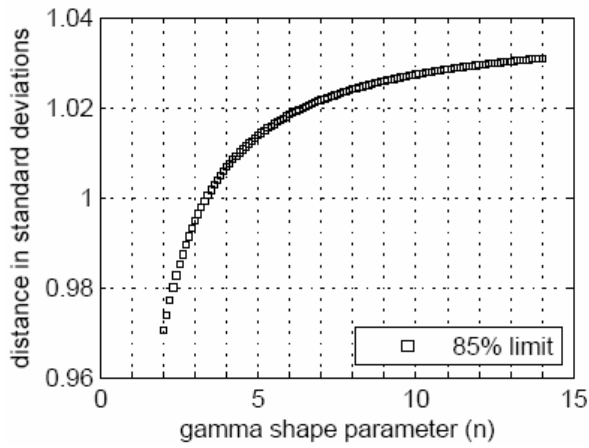


Fig 2. Distance (γ_{r1}) in standard deviations between first order cut-off point and mean, versus gamma shape parameter; (left) power generator runs above its rated capacity on 15% of occasions, (right) on 20% of occasions.

Fig 1 and 2 suggest a polynomial relationship between distance (γ_{r1}) and distribution shape parameter (n). Using a process of best-fit to find suitable polynomials, the author has arrived at the following set of functions to describe (γ_{r1}).

For a 95% limit:

$$\gamma_{r1} = 0.0010686 n^2 - 0.028463 n + 1.9782 \quad (19)$$

For a 90% limit:

$$\gamma_{r1} = -5.145 \times 10^{-6} n^4 + 0.000187 n^3 - 0.002381 n^2 + 0.010764 n + 1.3246 \quad (20)$$

For an 85% limit:

$$\gamma_{r1} = 8.5262 \times 10^{-5} n^3 - 0.002598 n^2 + 0.026785 n + 0.93363 \quad (21)$$

For an 80% limit:

$$\gamma_{r1} = 0.00011973 n^3 - 0.0037243 n^2 + 0.040337 n + 0.64533 \quad (22)$$

Functions (19) thru (22) are valid for shape parameters within the range ($2 \leq n \leq 14$) and should be substituted into (5), (6) and (18). For example, a result, as applied to a 90% generator limit is depicted in Fig 3.

III. CONCLUSION

The method of assessing the dependability of prospective wave energy conversion sites proposed in [2] has been extended to third order power distributions, such as those arising from wind speed data. More accurate expressions to describe the capacity factor of both wave height and wind speed data have also been developed. As was previously the case, these refined expressions are solely determined by the order of their moment and the gamma shape parameter (n).

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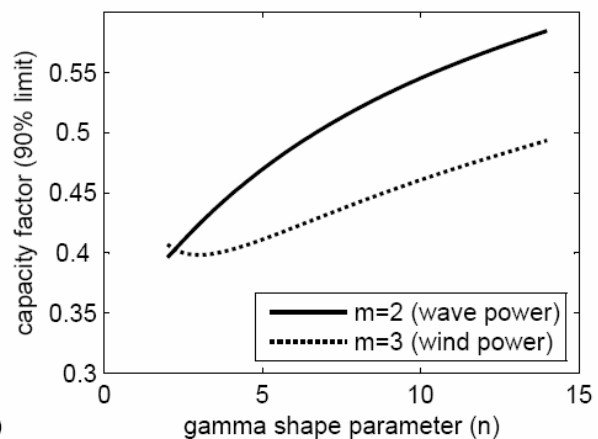
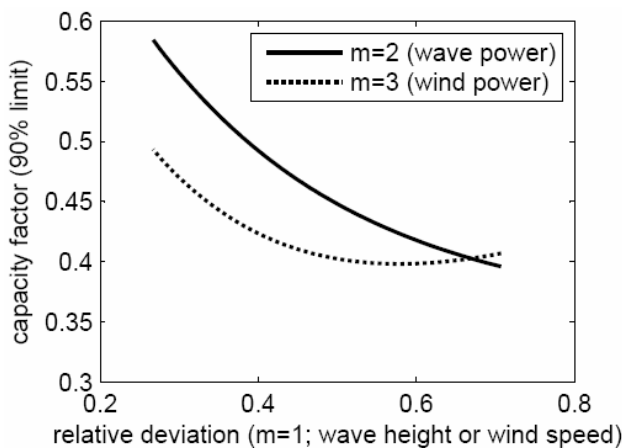


Fig 3. (left) capacity factor as a function of relative deviation of wave height or wind speed; (right) capacity factor as a function of gamma distribution shape parameter.